

VISCOUS GENERALIZED CHAPLYGIN GAS

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Viscous GCG(generalized Chaplygin gas) cosmology is discussed, assuming that there is bulk viscosity in the linear barotropic fluid and GCG. The dynamical analysis indicates that the phase $w_g = -1 + \sqrt{3}\gamma\kappa\tau_g/(\gamma - \sqrt{3}\kappa\tau_\gamma)$ is a dynamical attractor and the equation of state of GCG approaches it from either $w_g > -1$ or $w_g < -1$ depending on the choice of its initial cosmic density parameter and the ratio of pressure to critical energy density. Obviously, the equation of state w_g can cross the boundary $w_g = -1$. Also, from the point of view of dynamics, the parameters of viscous GCG should be in the range of $\gamma > \sqrt{3}\kappa\tau_\gamma/(1 - \sqrt{3}\kappa\tau_g)$ and $0 < \alpha < 1 + \sqrt{3}\kappa\gamma\tau_g/(\gamma - \sqrt{3}\kappa\tau_\gamma - \sqrt{3}\kappa\gamma\tau_g)$.

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1. Introduction

CMB anisotropy, supernovae and galaxies clustering strongly indicate that our universe is spatially flat, with two thirds of the energy content consisting of dark energy, a substance with negative pressure which can make the universe expand in an accelerating fashion. Present observation data constrain the range of the equation of state of dark energy as $-1.38 < w < -0.82$ [1], which indicates the possibility of dark energy with $w < -1$. If this is so, the universe can have some strange properties such as a future finite singularity which has been dubbed Big Rip[2] or Big Smash[3]. Proposed candidates for dark energy include the cosmological constant, quintessence with a single field[4] or with N coupled fields[5], phantom field with canonical[6] or Born-Infeld type Lagrangian[7], k -essence[8] and generalized Chaplygin gas(GCG)[9] which is based on the Chaplygin gas[10]. In particular, we have extended the equation of state of GCG to the regime $w < -1$ regime and shown that the GCG parameter is constrained by the dynamics to lie in the range $0 < \alpha < 1$ [11].

On the other hand, the role of dissipative processes has been conscientiously studied[12]. Dissipative effects, including both bulk and shear viscosity, play a very important role in the evolution of the universe. The viscosity theory of relativistic fluids was first suggested by Eckart[13] and Landau and Lifshitz[14], who considered only first-order deviation from equilibrium, which lead to parabolic differential equations and hence to an infinite speed of propagation for heat flow and viscosity, in contradiction with the principle of causality. The relativistic second-order theory was founded by Israel[15] and developed by Israel and Stewart[16], and has also been used in the evolution of the early universe[17]. However, the character of the evolution equation is very complicated in the framework of the full causal theory. Therefore, the conventional theory[14] is still applied to phenomena which are quasi-stationary, i.e., slowly varying on space and time scales characterized by the mean free path and the mean collision time. In the case of isotropic and homogeneous cosmologies, the dissipative process can be modelled as a bulk viscosity ζ within a thermodynamical approach. The shear viscosity η will be neglected, which is consistent with the usual practice[18]. The bulk viscosity introduces dissipation by only redefining the effective pressure, p_{eff} , according to $p_{eff} = p - 3\zeta H$ where ζ is the bulk viscosity coefficient and H is the Hubble parameter. The condition $\zeta > 0$ assures a positive entropy production in conformity with the second law of thermodynamics[19]. We are interested in the case $\zeta = \sqrt{3}\kappa^{-1}\tau H$, where τ is a constant. This assumption implicates that ζ is directly proportional to the divergence of the cosmic fluid's velocity vector. Therefore, it is physically natural, and has been considered previously in an astrophysical context[20].

In the present paper, we consider a viscous GCG cosmological model for the expanding universe, assuming that there is bulk viscosity in the linear barotropic fluid and GCG. The dynamical analysis indicates that the phase $w_g = -1 + \sqrt{3}\gamma\kappa\tau_g/(\gamma - \sqrt{3}\kappa\tau_\gamma)$ is a dynamical attractor and the equation of state of GCG approaches it from either $w_g > -1$ or $w_g < -1$ depending on the choice of its initial cosmic density parameter and the ratio of pressure to critical energy density, where we assume $\sqrt{3}\kappa\tau_g$ and $\sqrt{3}\kappa\tau_\gamma$ are small compared to 1. If indeed dark energy with

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$w < -1$ is within the regime of possibilities, then it would seem inevitable to inquire about a transition from $w < -1$ to $w > -1$, although a generalized scalar field cannot safely traverse $w = -1$. It is clear that w can cross the boundary $w_g = -1$ in the viscous GCG cosmology. Also, we have used cosmological dynamics to place constraints on some of the parameters.

2. Viscous GCG

GCG has a very simple equation of state, $p_g = -M^{4(\alpha+1)}/\rho_g^\alpha$, which yields an analytically solvable cosmological dynamics if the universe is GCG dominated. Another advantage of introducing GCG is to unify dark energy and dark matter into one equation of state, also known as quartessence[21]. However, detailed numerical analysis turns out to disfavor the dark matter modelled by the GCG equation of state[22]. But no observation has so far ruled out the possibility of GCG as dark energy. Therefore, it is quite possible that our universe contains a dark energy component modelled by the GCG as well as another linear barotropic fluid component with the equation of state $p_\gamma = (\gamma - 1)\rho_\gamma$. However, in this section, we focus first on the viscous GCG system, as the cosmological dynamics are analytically solvable if GCG is dominant. In the flat FRW universe, the field equations are

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\rho_g \quad (1)$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho_g + 3p_g - 9H\zeta) \quad (2)$$

The conservation equation is

$$\dot{\rho}_g + 3H(\rho_g + p_g - 3H\zeta) = 0 \quad (3)$$

Using the GCG equation of state, we obtain

$$\frac{a}{3} \frac{d\rho_g}{da} + \rho_g - \frac{M^{4(\alpha+1)}}{\rho_g^\alpha} - \sqrt{3}\kappa\rho_g^{\frac{1}{2}}\zeta = 0 \quad (4)$$

We shall be interested in the evolution of the late universe, from $t = t_0$ onwards. From Eq.(4), we have

$$a = a_0 \left(\exp \left[\int_{\rho_0}^{\rho_g} \frac{\rho^\alpha d\rho}{M^{4(\alpha+1)} + \sqrt{3}\kappa\rho^{\alpha+\frac{1}{2}}\zeta(\rho) - \rho^{\alpha+1}} \right] \right)^{\frac{1}{3}} \quad (5)$$

which is the general relation between the cosmological scale factor a and the energy density ρ_g .

In what follows we investigate two different solvable cases. If we choose $\zeta = \tau_g\sqrt{\rho}$, the energy density is given by

$$\rho_g = \left[\frac{M^{4(\alpha+1)}}{1 - \sqrt{3}\kappa\tau_g} + \left(\rho_0^{\alpha+1} - \frac{M^{4(\alpha+1)}}{1 - \sqrt{3}\kappa\tau_g} \right) \left(\frac{a_0}{a} \right)^{3(\alpha+1)(1-\sqrt{3}\kappa\tau_g)} \right]^{\frac{1}{\alpha+1}} \quad (6)$$

Obviously, $|p_g| \rightarrow \infty$ or 0 will not happen when $a \rightarrow a_s \neq 0$, corresponding to no future singularities. As $a \rightarrow \infty$, we have $w_g \rightarrow -(1 - \sqrt{3}\kappa\tau_g) > -1$.

As a second case, if we take $\zeta = \tau_g\rho^{\alpha+\frac{3}{2}}$ then Eq.(4) reads

$$\frac{a}{3(\alpha+1)} \frac{d}{da}(\rho_g^{\alpha+1}) = -\rho_g^{\alpha+1} + M^{4(\alpha+1)} + \sqrt{3}\kappa\tau_g\rho_g^{2(\alpha+1)} \quad (7)$$

and if $\tau_g < (4\sqrt{3}\kappa M^{4(\alpha+1)})^{-1}$, it has two stationary points at $\rho_g = \left(\frac{1-\Delta}{2\sqrt{3}\kappa\tau_g} \right)^{\frac{1}{\alpha+1}} \equiv \rho_1$ and $\rho_g = \left(\frac{1+\Delta}{2\sqrt{3}\kappa\tau_g} \right)^{\frac{1}{\alpha+1}} \equiv \rho_2$ where $\Delta = (1 - 4\sqrt{3}\kappa\tau_g M^{4(\alpha+1)})^{\frac{1}{2}}$. The first stationary point is an attractor, and corresponds to $a \rightarrow \infty$. The second stationary point is unstable, and corresponds to $a = 0$. If $\rho_0 < \rho_1$ or $\rho_1 < \rho_0 < \rho_2$ then the model evolves As $a \rightarrow \infty$, $\rho_g \rightarrow \rho_1$ and $w_g \rightarrow -\frac{2\sqrt{3}\kappa\tau_g M^{4(\alpha+1)}}{1-\Delta} > -1$. However, if $\rho_0 > \rho_2$ then as $a \rightarrow a_0 \left(\frac{\rho_0^{\alpha+1} - \rho_1^{\alpha+1}}{\rho_0^{\alpha+1} - \rho_2^{\alpha+1}} \right)^{\frac{1}{3\Delta(\alpha+1)}}$, $\rho_g \rightarrow \infty$ and

$p_g \rightarrow \infty$ so the model encounters a Type III singularity (where the pressure and density diverge in finite proper time but the metric remains non-singular).

3. Autonomous system

A general study of the phase space system of quintessence and phantom in FRW universe has been given in Refs.[23, 24]. For the viscous GCG cosmological dynamical system, the corresponding equation of motion and Einstein equations can be written as

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_\gamma + p_\gamma - 3\zeta_\gamma H + \rho_g + p_g - 3\zeta_g H) \quad (8)$$

$$\dot{\rho}_\gamma = -3H(\rho_\gamma + p_\gamma - 3\zeta_\gamma H) \quad (9)$$

$$\dot{\rho}_g = -3H(\rho_g + p_g - 3\zeta_g H) \quad (10)$$

$$H^2 = \frac{\kappa^2}{3}(\rho_\gamma + \rho_g) \quad (11)$$

where ρ_γ is the density of fluid with a barotropic equation of state $p_\gamma = (\gamma - 1)\rho_\gamma$, and $0 \leq \gamma \leq 2$ is a constant that relates to the equation of state by $w_\gamma = \gamma - 1$; ρ_g and p_g are the energy density and pressure of GCG, respectively. The overdot represents the derivative with respect to cosmic time t . We choose $\zeta_g = \tau_g \sqrt{\rho} = \sqrt{3}\kappa^{-1}\tau_g H$ and $\zeta_\gamma = \tau_\gamma \sqrt{\rho} = \sqrt{3}\kappa^{-1}\tau_\gamma H$.

To analyze the dynamical system, we rewrite the equations with the following dimensionless variables:

$$\begin{aligned} x &= \frac{\kappa^2 \rho_g}{3H^2} \\ y &= \frac{\kappa^2 p_g}{3H^2} \\ N &= \ln a \end{aligned} \quad (12)$$

The dynamical system can be reduced to

$$\frac{dx}{dN} = -3(x + y) + 3\sqrt{3}\kappa\tau_g + 3x[\gamma(1 - x) + x + y] - 3\sqrt{3}\kappa(\tau_g + \tau_\gamma)x \quad (13)$$

$$\frac{dy}{dN} = 3\alpha(y + \frac{y^2}{x}) - 3\sqrt{3}\alpha\kappa\tau_g \frac{y}{x} + 3y[\gamma(1 - x) + x + y] - 3\sqrt{3}\kappa(\tau_g + \tau_\gamma)y \quad (14)$$

From Eq.(11), we have

$$\Omega_g + \Omega_\gamma = 1 \quad (15)$$

where $\Omega_g \equiv x$ and $\Omega_\gamma \equiv \frac{\kappa^2 \rho_\gamma}{3H^2}$ are the cosmic density parameters for GCG and linear barotropic fluid, respectively. The equation of state can be expressed in terms of the new variables as

$$w_g = \frac{p_g}{\rho_g} = \frac{y}{x} \quad (16)$$

and the sound speed is

$$c_s^2 = -\alpha \frac{y}{x} \quad (17)$$

In the following we need to consider the two cases of $\gamma = 1$ and $\gamma \neq 1$. In the case of $\gamma \neq 1$, the critical points of the system are

$$(x^{(1)}, y^{(1)}) = \left(\frac{\gamma - 1 - \sqrt{3}\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma - \Sigma}{2(\gamma - 1)}, 0 \right) \quad (18)$$

$$(x^{(2)}, y^{(2)}) = \left(\frac{\gamma - 1 - \sqrt{3}\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma + \Sigma}{2(\gamma - 1)}, 0 \right) \quad (19)$$

and

$$(x^{(3)}, y^{(3)}) = \left(1 - \frac{\sqrt{3}\kappa\tau_\gamma}{\gamma}, -1 + \sqrt{3}\kappa\tau_g + \frac{\sqrt{3}\kappa\tau_\gamma}{\gamma} \right) \quad (20)$$

which correspond to the linear barotropic fluid-dominated phase, GCG matter-dominated phase and GCG vacuum-energy-dominated phase, respectively. Here

$$\Sigma = [4\sqrt{3}(\gamma - 1)\kappa\tau_g + (\gamma - 1 - \sqrt{3}\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma)^2]^{\frac{1}{2}} \quad (21)$$

If we linearize the system near the critical points $(x^{(i)}, y^{(i)})$, $i = 1, 2, 3$ and then translate the system to the origin, we can readily write the first order perturbation equation as

$$\frac{dU}{dN} = A^{(i)}U \quad (22)$$

where U is a 2-column vector consisting of the perturbations of x and y . $A^{(i)}$ is a 2×2 matrix for the critical point $(x^{(i)}, y^{(i)})$. The stability of the critical points is determined by the eigenvalues of the matrix $A^{(i)}$ at the critical point $(x^{(i)}, y^{(i)})$. For the point $(x^{(1)}, y^{(1)})$, the two eigenvalues are

$$\begin{aligned} \lambda_1^{(1)} &= 3\Sigma \\ \lambda_2^{(1)} &= \frac{3(1 + \alpha)(\gamma - 1 + \sqrt{3}\kappa\tau_g - 2\sqrt{3}\gamma\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma - \Sigma)}{\gamma - 1 - \sqrt{3}\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma - \Sigma} \end{aligned} \quad (23)$$

Obviously, the linear barotropic fluid-dominated phase is unstable, and so evolves to GCG-dominated phase. For the point $(x^{(2)}, y^{(2)})$ the two eigenvalues are

$$\begin{aligned} \lambda_1^{(2)} &= -3\Sigma \\ \lambda_2^{(2)} &= \frac{3(1 + \alpha)(\gamma - 1 + \sqrt{3}\kappa\tau_g - 2\sqrt{3}\gamma\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma + \Sigma)}{\gamma - 1 - \sqrt{3}\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma + \Sigma} \end{aligned} \quad (24)$$

It can be shown that $\lambda_2^{(2)} > 0$, and so the point is stable if $\alpha > -1$ and $\gamma < \frac{\sqrt{3}\kappa\tau_\gamma}{1 - \sqrt{3}\kappa\tau_g}$. However, since we want the GCG to first behave as matter and then evolve to behave like dark energy, it will not be appropriate if $(x^{(2)}, y^{(2)})$ corresponds to a stable attractor phase. In other words $\gamma < \frac{\sqrt{3}\kappa\tau_\gamma}{1 - \sqrt{3}\kappa\tau_g}$ should not be considered in the real models. For the critical point $(x^{(3)}, y^{(3)})$, the corresponding eigenvalues of matrix $A^{(3)}$ are

$$\begin{aligned} \lambda_1^{(3)} &= -\frac{3}{2(\gamma - \sqrt{3}\kappa\tau_\gamma)}(\Theta + \Xi) \\ \lambda_2^{(3)} &= -\frac{3}{2(\gamma - \sqrt{3}\kappa\tau_\gamma)}(\Theta - \Xi) \end{aligned} \quad (25)$$

where

$$\begin{aligned} \Theta &= \gamma^2 + [(1 + \alpha)(1 - \sqrt{3}\kappa\tau_g) - 2\sqrt{3}\kappa\tau_\gamma]\gamma - (1 + \alpha)\sqrt{3}\kappa\tau_\gamma + 3\kappa^2(\tau_g\tau_\gamma + \tau_\gamma^2) \\ \Xi &= \left(-4(1 + \alpha)(\gamma - \sqrt{3}\kappa\tau_g)^2[\gamma(1 - \sqrt{3}\kappa\tau_g) - \sqrt{3}\kappa\tau_\gamma] \right. \\ &\quad \left. + [\gamma^2 + [(1 + \alpha)(1 - \sqrt{3}\kappa\tau_g) - 2\sqrt{3}\kappa\tau_\gamma]\gamma - (1 + \alpha)\sqrt{3}\kappa\tau_\gamma + 3\kappa^2(\tau_g\tau_\gamma + \tau_\gamma^2)]^2 \right)^{\frac{1}{2}} \end{aligned} \quad (26)$$

TABLE I: The initial values of x and y in the plots FIG.1-FIG.3

x	0.14	0.15	0.16	0.17	0.18	0.18
y	-0.19	-0.18	-0.17	-0.16	-0.15	-0.14

TABLE II: The initial values of x and y in the plot FIG.4

x	0.0140	0.0240	0.0340	0.0440	0.0540
y	-0.0042	-0.0036	-0.0030	-0.0024	-0.0018

Note that $\lambda_1^{(3)}\lambda_2^{(3)} = 9(\alpha + 1)(\gamma - \sqrt{3}\gamma\kappa\tau_g - \sqrt{3}\kappa\tau_\gamma)$, and it can be shown that $(x^{(3)}, y^{(3)})$ is stable for $\alpha > -1$ and $\gamma > \frac{\sqrt{3}\kappa\tau_\gamma}{1 - \sqrt{3}\kappa\tau_g}$.

In the case of $\gamma = 1$, the system has only two critical points, the first one is $(x^{(*)}, y^{(*)}) = \left(\frac{\tau_g}{\tau_g + \tau_\gamma}, 0\right)$, the second one is still Eq.(20) with everywhere replaced by $\gamma = 1$. At the first critical point, the eigenvalues are $\lambda_1^{(*)} = -3\sqrt{3}\kappa(\tau_g + \tau_\gamma)$ and $\lambda_2^{(*)} = 3(\alpha + 1)(1 - \sqrt{3}\kappa(\tau_g + \tau_\gamma))$, and so the point is unstable if $\alpha > -1$ and $1 - \sqrt{3}\kappa(\tau_g + \tau_\gamma) > 0$. It can be shown that the second critical point is stable under the same assumptions. So we obtain the same stable point in both cases. This critical point corresponds to a GCG-dominated phase

$$\Omega_g = 1 - \frac{\sqrt{3}\kappa\tau_\gamma}{\gamma}, \Omega_\gamma = \frac{\sqrt{3}\kappa\tau_\gamma}{\gamma} \quad (27)$$

and its equation of state is

$$w_g = -1 + \frac{\sqrt{3}\gamma\kappa\tau_g}{\gamma - \sqrt{3}\kappa\tau_\gamma} \quad (28)$$

In the $\tau_\gamma = \tau_g = 0$ case, we have $\Omega_g = 1$ and $w_g = -1$ which is a late time de Sitter attractor[21]. In the $\tau_\gamma = 0$ and $\tau_g \neq 0$ case, this analysis is consistent with the results of exact solution (6).

4. Numerical analysis

Next, we study the above dynamical system numerically. For definiteness, we choose the parameters to be $\gamma = 1$ and $\alpha = 0.5$. The initial values of x and y are chosen as shown in TABLE I and the results are contained in FIG.1-FIG.3. From FIG.1, we can observe that all orbits tend to an attractor which corresponds to $w_g = -\frac{91}{97}$. From FIG.2, we can observe that for different initial values of ρ_g and p_g , the equation of state w_g approaches the attractor $w_g = -\frac{91}{97}$ from either $w_g > -1$ or $w_g < -1$. Irrespective of whether the initial choice has $w_g > -1$ or $w_g < -1$, the equation of state will eventually mimic that of quintessence in the viscous cosmology. It is worth noting that if we choose the parameters so that the GCG behaves as phantom, it is no longer possible to make it behave as matter at an early epoch unless the viscosity coefficients are anomalously large. However, in our setup of this paper, we have included a linear barotropic fluid that could be used to mimic the matter sector of our universe and thus GCG can be considered only as dark energy. The evolution of the sound speed c_s^2 is shown in FIG.3, where c_s^2 tends to $\frac{91}{194}$ for different initial values of ρ_g and p_g , and $\sqrt{3}\kappa\tau_\gamma = 0.03$, $\sqrt{3}\kappa\tau_g = 0.06$.

The initial values of x and y are chosen as shown in TABLE II and the results are contained in FIG.4. The different values of the parameters τ_γ and τ_g are chosen as shown in TABLE III and the results are contained in FIG.5.

TABLE III: The parameter values of τ_g and τ_γ

$\sqrt{3}\kappa\tau_g$	0.02	0.04	0.06	0.08	0.10	0.12
$\sqrt{3}\kappa\tau_\gamma$	0.01	0.02	0.03	0.04	0.05	0.06

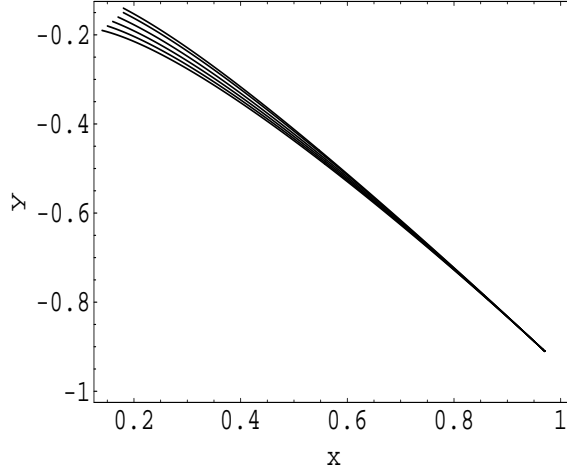


FIG. 1: The phase diagram of the viscous GCG system in terms of x and y for different initial values of x and y , and $\sqrt{3}\kappa\tau_g = 0.06$, $\sqrt{3}\kappa\tau_\gamma = 0.03$.

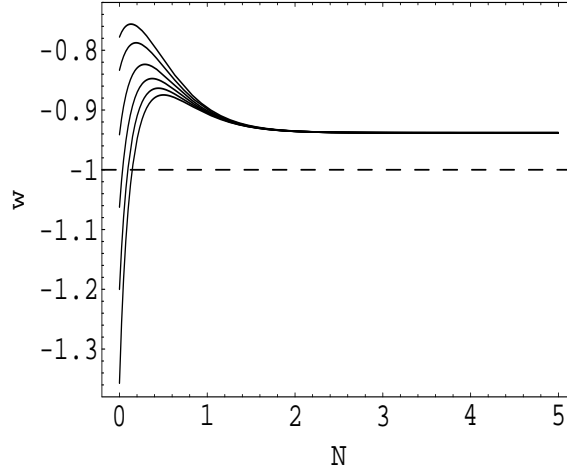


FIG. 2: The evolution of the equation of state of viscous GCG for different initial values of x and y , where we have taken $\sqrt{3}\kappa\tau_g = 0.06$, $\sqrt{3}\kappa\tau_\gamma = 0.03$. The curves from bottom to top correspond to the initial conditions specified in Table I from left to right respectively.

5. Conclusions

In the present paper, we have analyzed the dynamical evolution of viscous GCG for different parameters and initial conditions. Our results are as follows:

(1) We have shown that different initial values of ρ_g and p_g will lead to different tracks ($w_g > -1$ and $w_g < -1$) for the equation of state w_g as it approaches the dynamical attractor phase $w_g = -1 + \frac{\sqrt{3}\gamma\kappa\tau_g}{\gamma - \sqrt{3}\kappa\tau_\gamma}$. That is to say, the comparison of theoretical values to observational ones places a constraint on the bulk viscosity coefficient. Recent astrophysical data indicate that the effective equation of state parameter w_{eff} lies in the interval: $-1.38 < w_{eff} < -0.82$ [1], so that we have the constraint $\sqrt{3}\kappa(\frac{50}{9}\tau_g + \tau_\gamma) < 1$.

(2) We can also use the requirement that $0 < c_s^2 < 1$ in the GCG to place the constraint $0 < \alpha < 1 + \frac{\sqrt{3}\kappa\gamma\tau_g}{\gamma - \sqrt{3}\kappa\tau_\gamma - \sqrt{3}\kappa\gamma\tau_g}$ on the parameter α in the model.

(3) The equation of state w_g can cross the boundary $w_g = -1$.

(4) From the point of view of the dynamics, the bulk viscosity coefficient should satisfy $\gamma > \frac{\sqrt{3}\kappa\tau_\gamma}{1 - \sqrt{3}\kappa\tau_g}$.

(5) In the viscous model, the ratio of cosmic density parameters $\frac{\Omega_\gamma}{\Omega_g}$ approaches the constant $\frac{\sqrt{3}\kappa\tau_\gamma}{\gamma - \sqrt{3}\kappa\tau_\gamma}$. If we suppose

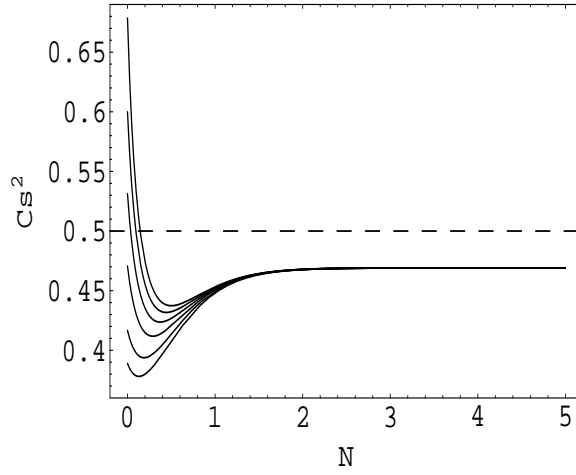


FIG. 3: The evolution of the sound speed c_s^2 for different initial values of x and y in a viscous cosmological model with $\sqrt{3}\kappa\tau_g = 0.06$, $\sqrt{3}\kappa\tau_\gamma = 0.03$. The curves from top to bottom correspond to the initial conditions specified in Table I from left to right respectively.

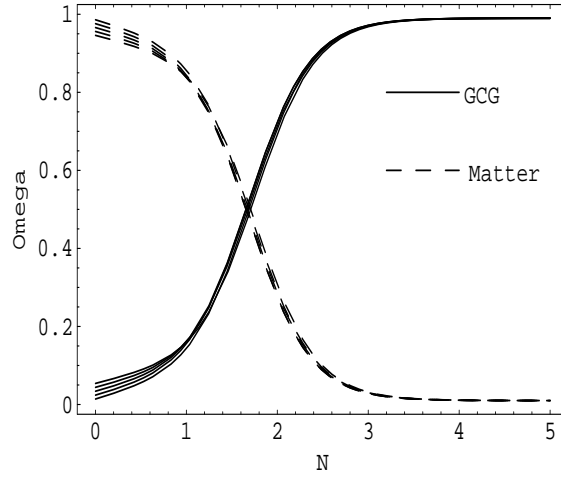


FIG. 4: The evolution of the cosmic density parameter for matter Ω_γ and Ω_g respectively at different initial values of x and y , and $\sqrt{3}\kappa\tau_g = 0.02$, $\sqrt{3}\kappa\tau_\gamma = 0.01$.

the present universe is in the epoch of $\frac{\Omega_\gamma}{\Omega_g}$ approximating to a constant, we can obtain the constraints $\sqrt{3}\kappa\tau_\gamma = 0.3\gamma$ and $\sqrt{3}\kappa\tau_g < 0.7$.

Finally, it is worth noting that as a phenomenological model of the evolution of the late universe, it is reasonable for our model to leave the causal viscosity theory out of account.

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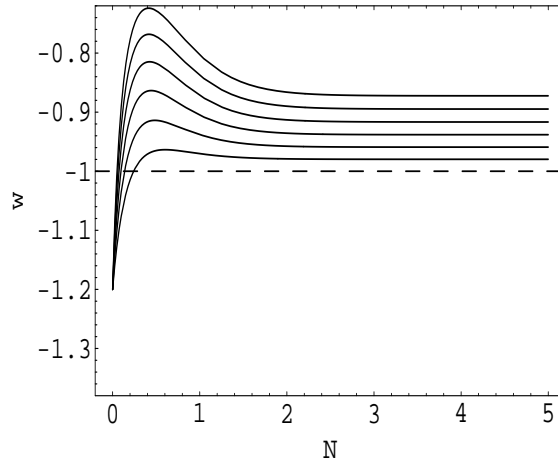


FIG. 5: The evolution of the equation of state of viscous GCG for different bulk viscosity coefficients, where we have taken $x = 0.15$ and $y = -0.18$. The curves from bottom to top correspond to the parameter values specified in Table 3 from left to right, respectively.

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